

Learning Targets:

- Write, solve, and graph absolute value equations.
- Solve and graph absolute value inequalities.

SUGGESTED LEARNING STRATEGIES: Marking the Text, Interactive Word Wall, Close Reading, Create Representations, Think-Pair-Share, Identify a Subtask, Quickwrite, Self Revision/Peer Revision

You can use the definition of absolute value to solve *absolute value equations* algebraically.

Since

$$|ax + b| = \begin{cases} -(ax + b) & \text{if } ax + b < 0 \\ ax + b & \text{if } ax + b \geq 0 \end{cases}$$

then the equation $|ax + b| = c$ is equivalent to $-(ax + b) = c$ or $(ax + b) = c$.

Since $-(ax + b) = c$ is equivalent to $ax + b = -c$, the absolute value equation $|ax + b| = c$ is equivalent to $ax + b = -c$ or $ax + b = c$.

Example A

Solve $2|x - 1| - 5 = 1$. Graph the solutions on a number line.

Step 1: Isolate the absolute value expression. Add 5 to both sides and then divide by 2.

$$\begin{aligned} 2|x - 1| - 5 &= 1 \\ 2|x - 1| &= 6 \\ |x - 1| &= 3 \end{aligned}$$

Step 2: Write and solve two equations using the definition of absolute value.

$$\begin{aligned} x - 1 &= 3 & \text{or} & & x - 1 &= -3 \\ x &= 4 & \text{or} & & x &= -2 \end{aligned}$$

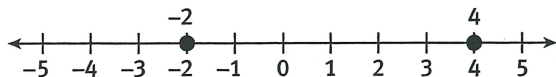
Solve each mini.

Solution: There are two solutions: $x = 4$ and $x = -2$

Check to see if both solutions satisfy the original equation. Substitute 4 and -2 for x in the original equation.

$$\begin{aligned} 2|4 - 1| - 5 &= 1 & 2|-2 - 1| - 5 &= 1 \\ 2|3| - 5 &= 1 & 2|-3| - 5 &= 1 \\ 2(3) - 5 &= 1 & 2(3) - 5 &= 1 \\ 6 - 5 &= 1 & 6 - 5 &= 1 \end{aligned}$$

To graph the solutions, plot points at 4 and -2 on a number line.



Try These A

Solve each absolute value equation. Graph the solutions on a number line.

a. $|x - 2| = 3$

$$\begin{aligned} x - 2 &= 3 & x - 2 &= -3 \\ x &= 5 & x &= -1 \end{aligned}$$

c. $|x - 3| + 4 = 4$

$$\begin{aligned} -4 - 4 \\ |x - 3| &= 0 \\ x - 3 &= 0 \\ x &= 3 \end{aligned}$$

b. $|x + 1| - 4 = -2$

$$\begin{aligned} x + 1 &= 2 & x + 1 &= -2 \\ x &= 1 & x &= -3 \end{aligned}$$

d. $|x + 2| + 3 = 1$

$$\begin{aligned} -3 - 3 \\ |x + 2| &= -2 \end{aligned}$$

no solution!

A distance cannot be negative!

My Notes

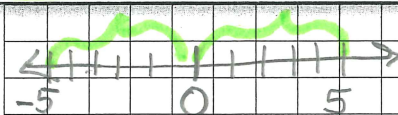
MATH TERMS

An **absolute value equation** is an equation involving the absolute value of a variable expression.

MATH TIP

Recall that the geometric interpretation of $|x|$ is the distance from the number x to 0 on a number line.

If $|x| = 5$, then $x = -5$ or $x = 5$ because those two values are both 5 units away from 0 on a number line.



Two numbers 5 and -5 have a distance of 5 from zero on the # line!

My Notes

1. **Reason abstractly.** How many solutions are possible for an absolute value equation having the form $|ax + b| = c$, where a , b , and c are real numbers?

Example B

The temperature of the wave pool at Sapphire Island can vary up to 4.5°F from the target temperature of 82°F . Write and solve an absolute value equation to find the temperature extremes of the wave pool. (The temperature extremes are the least and greatest possible temperatures.)

Step 1: Write an absolute value equation to represent the situation.

Let t represent the temperature extremes of the wave pool in degrees Fahrenheit.

$$|t - 82| = 4.5$$

Step 2: Use the definition of absolute value to solve for t .

$$|t - 82| = 4.5$$

$$t - 82 = 4.5 \quad \text{or} \quad t - 82 = -4.5$$

$$t = 86.5 \quad \text{or} \quad t = 77.5$$

solve each mini.

2 solutions

Solution: The greatest possible temperature of the wave pool is 86.5°F , and the least possible temperature is 77.5°F . Both of these temperatures are 4.5°F from the target temperature of 82°F .

Try These B

The pH of water is a measure of its acidity. The pH of the water on the Seal Slide can vary up to 0.3 from the target pH of 7.5. Use this information for parts a–c.

- a. Write an absolute value equation that can be used to find the extreme pH values of the water on the Seal Slide. Be sure to explain what the variable represents.

$$|x - 7.5| = 0.3$$

x = min + max pH levels

- b. Solve your equation, and interpret the solutions.

$$x - 7.5 = 0.3 \quad \text{or} \quad x - 7.5 = -0.3$$

$$x = 7.8 \quad \text{or} \quad x = 7.2$$

- c. **Reason quantitatively.** Justify the reasonableness of your answer to part b.

These two answers are each 0.3 away from 7.5 on the # line.

MATH TIP

You know that the distance from t to 82°F on a thermometer is 4.5°F . This distance can be modeled with the absolute value expression $|t - 82|$.